

■ The Poster *The Mathematical Functions of Mathematica*

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Wolfram Research Inc. produced a unique media—five large posters (more than 100 sq. ft.) which describe the world of elementary and special functions supported by the technical computing system *Mathematica*. Starting from the elementary functions and going to Bessel functions, orthogonal polynomials, hypergeometric functions (including MeijerG function, Appell F_1 function, ...), elliptic functions (including Jacobi elliptic functions, ...) the poster leads up to function of number theory (Jacobi symbols, ...).

The set of posters has been developed at Wolfram Research, Inc. by Oleg Marichev and Michael Trott, David Gehrig and Andy Hunt.

Each poster consists primarily of a set of hierarchically distributed boxes, each containing unique information about one specific function. This information includes notations, definitions, series-, product-, integral-, and continued fraction representations, functional and differential equations, main theorems, important applications and a short history. Every function is visualized with a colorful graphic. The posters also include a picture and biographical data of the person who "invented" the function. The poster embraces different pieces of information and represents them in a coherent, ordered, and visually pleasing way. Hundreds new formulas were derived and presented on the poster. For example, the following relations for Sqrt and ArcCos from Cos take place:

$$\sqrt{z^2} = \sqrt{i z} \sqrt{-i z}$$

$$\cos^{-1}(\cos(z)) = \frac{\pi}{2} \left(1 - (-1)^{\lfloor -\frac{\text{Re}(z)}{\pi} \rfloor} \right) + (-1)^{\lfloor -\frac{\text{Re}(z)}{\pi} \rfloor} \left((1 + (-1)^{\lfloor \frac{\text{Re}(z)}{\pi} \rfloor + \lfloor -\frac{\text{Re}(z)}{\pi} \rfloor}) \theta(\text{Im}(z)) - 1 \right) \left(z + \pi \left[-\frac{\text{Re}(z)}{\pi} \right] \right).$$

Instead of three separate asymptotical formulas which described known Stockes phenomenon for AiryBi function we can use just one universal formula which is correct for all sectors in complex plane:

$$\text{Bi}(z) \sim \frac{(-z^3)^{-5/12}}{2\sqrt{\pi}} \left((-1)^{5/12} e^{-\frac{2i}{3}\sqrt{-z^3}} \left(\frac{1}{\sqrt[3]{-1}} \sqrt[3]{-z^3} + z \right) {}_2F_0\left(\frac{1}{6}, \frac{5}{6}; ; \frac{3i}{4\sqrt{-z^3}}\right) - \right. \\ \left. (-1)^{7/12} e^{\frac{2i}{3}\sqrt{-z^3}} \left(\sqrt[3]{-1} \sqrt[3]{-z^3} + z \right) {}_2F_0\left(\frac{1}{6}, \frac{5}{6}; ; -\frac{3i}{4\sqrt{-z^3}}\right) \right) /; |z| \rightarrow \infty$$

It was not our primary intention to make a complete classification of mathematical functions, however the functions on a poster are arranged out in a well organized way, the most general function on top and their special cases on the bottom. The five parts of the poster-set are

- 1.) Elliptic Functions
- 2.) Elementary Functions and Bessel Functions
- 3.) Hypergeometric Functions
- 4.) Zeta and Mathieu, and Other Functions
- 5.) The Special Functions

The 5.) panel contains an instructive overview of the world of special functions, including the history of handbooks and tables. Many examples show how, using *Mathematica*, one can work constructively with special functions. In addition, uniform tables of all integrals and derivatives, as well as (if possible) hypergeometric ${}_pF_q$, and Meijer G representations of all functions are presented.

The poster set describes more than 250 functions with more than 5000 nicely typesetted formulas and more than 300 graphics. More than 600 connecting lines run in an organized way across the poster, indicating the relations between all functions.

For details see <http://www.specialfunctions.com>.