Test #1 **AMATYC Student Mathematics League** **October/November 2004**

1. What is the slope of a line parallel to the line with equation 2x - 5y = 10?

A. $\frac{2}{5}$ B. $\frac{-2}{5}$ C. $\frac{5}{2}$ D. $\frac{-5}{2}$ E. -2

2. In square ABCD, point E is between A and B, and point F is between B and C. Find the sum of the measures of $\angle$AEF and $\angle$EFC.

A. 90° B. 180° C. 270° D. 360° E. not determined

3. The letters of AMATYC are written as follows: Letters appear in increasing order of the number of line segments or arcs used to write them; identical letters do not appear consecutively. What is the required sequence?

4. A newspaper advertises that it sells the Sunday paper for one-third the price of the rest of the week's papers. If a weekly subscription costs between \$2.20 and \$2.30, what is the cost of one Sunday paper and one daily paper?

A. 56¢ B. 81¢ C. 84¢ D. 87¢ E. \$1.12

5. If h(x) = 2x - 8, find $h^{-1}(6)$. A. -4 B. 1/4 C. 7 D. 11 E. 20

6. A date is called *weird* if the number of its month and the number of its day have greatest common factor 1. What are the fewest number of *weird* days in any month?

A. 9 B. 10 C. 11 D. 14 E. 15

7. Lucia is not yet 80 years old. Each of her sons has as many sons as brothers. The combined number of Lucia's sons and grandsons equals her age, and her oldest grandson is 29. How old is Lucia? Place your numerical answer in the corresponding answer blank.

8. What is arccsc $\frac{5}{4}$ + arcsec $\frac{5}{4}$ + arccot $\frac{5}{4}$ + arccot $\frac{4}{5}$?

A. 2π B. π C. $\frac{\pi}{2}$ D. $\frac{\pi}{3}$ E. $\frac{\pi}{4}$

9. George bought groceries with a \$10 bill. The cost of the groceries had 3 different digits, and the amount of his change had the same 3 digits in a different order. What was the sum of the digits in the cost?

A. 13 B. 14 C. 15 D. 16 E. cannot be determined

10. Let N be the smallest number divisible by 33 which is greater than 1,000,000 and whose digits are all 0's and 1's. What are N's leading four digits?

A. 1001 B. 1010 C. 1011 D. 1101 E. 1110

11. In a recent competition, each of three teams played each other team once. In the table, GF is "goals for" (the number of goals scored by a team), and GA is "goals against" (the number of goals scored against a team). What was the score of the S vs J game (Give S's goals first)?

Team	Wins	Losses	Ties	GF	GA
S	1	0	1	6	4
F	1	0	1	3	2
J	0	2	0	2	5

A. 2-0 B. 2-1 C. 3-1 D. 3-2 E. 4-2

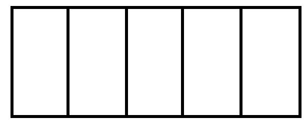
12. The song "What a Beautiful Life" has the lyric, "Day 18,253, well, honey, that's fifty years." If the lyric was supposed to be exactly correct, by how many days is it wrong?

A. 0 or 1 B. 2 to 4 C. 5 to 7 D. 8 to 10 E. 11 to 13

13. Chris traveled 1 hour longer and 2 miles farther than Calvin, but averaged 3 mph slower. If the sum of their times was 4 hours, what was the sum in miles of the distances they traveled?

A. 5 B. 26 C. 28.5 D. 30.5 E. 46

14. When five identical tables are placed end-to-end as on the left, the ratio of perimeter to area of the resulting shape is 1/2; when they are placed side-by-side as on the right, the ratio of perimeter to area is 3/10. What is the ratio of perimeter to area of one table?



A. 2/3 B. 3/4 C. 4/5 D. 6/5 E. 3/2

15. Find the sum of the x- and y-intercepts of the line with slope $\frac{-1}{3}$ which is the hypotenuse of a right triangular region in Quadrant I with legs the x- and y-axes and area $\frac{392}{3}$.

A. $\frac{28}{3}$ B. $\frac{56}{3}$ C. 28 D. $\frac{112}{3}$ E. $\frac{168}{3}$

16. Let A = {0,1,2,3,4,5,6,7,8,9}. How many three-element subsets of A contain at least two consecutive integers?

A. 32 B. 40 C. 48 D. 56 E. 64

17. If x, y, and z are positive integers with $x + 2y + 2z = 2005$ and $2x + 2y + z = 2004$, find the smallest possible value of $x + y + z$.

A. 999 B. 1000 C. 1001 D. 1002 E. 1003

18. A store has four open checkout stands. In how many ways could six customers line up at the checkout stands?

A. 210 B. 1296 C. 4096 D. 60480 E. 151200

19. Circle O has equation $x^2 + y^2 = 16$. If P is (1,0), Q is (-1,0), and R is any point on circle O, what is the largest possible value of PR + QR?

A. 8 B. $2\sqrt{17}$ C. $6\sqrt{2}$ D. 17/2 E. $4\sqrt{5}$

20. Suppose $f(x) = ax + b$, $g(x) = bx + a$ (a, b integers). If $f(1) = 8$ and $f(g(50)) - g(f(50)) = 28$, find the product of a and b.

A. 5 B. 12 C. 48 D. 182 E. 210

ANSWERS:

1.	C	2.	C	3.	CTAYAM	4.	C	5.	C
6.	B	7.	64	8.	B	9.	B	10.	D
11.	E	12.	D	13.	D	14.	A	15.	D
16.	E	17.	E	18.	D	19.	B	20.	B

Test #2 **AMATYC Student Mathematics League** **February/March 2005**

1. Hiromi buys a TV in Oregon (where there is no sales tax) and receives a 13% discount on the list price. Later she sees an ad offering a 19% discount. If the store agrees to refund the difference and Hiromi gets $21 back, what is the TV's list price?

A. \$300 B. \$320 C. \$325 D. \$330 E. \$350

2. What is the coefficient of x^2 in the expansion of $(x^2 + 3x - 1)^2$?

A. -2 B. -1 C. 2 D. 7 E. 9

3. Find the sum of the values of x for which $\frac{x - 2}{x^2 - 4x + 3}$ is undefined.

A. 3 B. 4 C. 5 D. 6 E. 7

4. The lines with equations ax + by = c and dx + ey = f are perpendicular (a, b, c, d, e, f constants). Which of the following must be true?

A. ad - be = 0 B. ad + be = -1 C. ae + bd = -1 D. ae + bd = 0 E. ad + be = 0

5. A palindrome is a word or a number (like RADAR or 1221) which reads the same forwards and backwards. If dates are written in the format MMDDYY, how many dates in the 21st century are palindromes?

A. 1 B. 12 C. 24 D. 36 E. 144

6. In square ABCD, E is the midpoint of CD. Suppose AE intersects BD at F and the extension of side BC at G. If AF = 2005 and EF = 1000, find EG.

A. 1000 B. 2000 C. 2005 D. 3005 E. 4010

7. For positive values of x for which $\mathrm{Sec}^{-1}(x)$ is in the first quadrant, $\mathrm{Sec}^{-1}(x) =$

A. $\frac{1}{\mathrm{Cos}^{-1}(x)}$ B. $\sec\left(\frac{1}{x}\right)$ C. $\cos x$ D. $\cos\left(\frac{1}{x}\right)$ E. $\mathrm{Cos}^{-1}\left(\frac{1}{x}\right)$

8. Mrs. Abbott finds that the number of possible groups of 3 students in her class is exactly five times the number of possible groups of 2 students. How many students are in her class?

A. 15 B. 17 C. 20 D. 22 E. 25

9. In how many ways can slashes be placed among the letters AMATYCSML to separate them into four groups with each group including at least one letter?

A. 28 B. 56 C. 70 D. 84 E. 112

10. Two motorists set out at the same time to go from Danbury to Norwich, 100 miles apart. They follow the same route and travel at different but constant speeds of an integral number of miles per hour. The difference in their speeds is a prime number of miles per hour, and after driving for two hours, the distance of the slower car from Danbury is five times that of the faster car from Norwich. What is the faster car's speed?

A. 40 mph B. 42 mph C. 44 mph D. 46 mph E. 48 mph

11. The sum cos 1° + cos 2° + cos 3° + … + cos 357° + cos 358° + cos 359° is equal to

A. $\frac{[illegible]}{2}$ B. [illegible] C. 0 D. 1 E. –1

12. If $M = \begin{bmatrix} 0 & 2 \\ -5 & 0 \end{bmatrix}$ and $N = \begin{bmatrix} 0 & 5 \\ -2 & 0 \end{bmatrix}$, find M^{2005}.

A. $10^{1002}M$ B. $10^{1002}N$ C. $10^{2004}M$ D. $10^{2004}N$ E. $10^{2005}M$

13. A basketball team scores 78 points on 41 baskets (field goals count 2 points, free throws 1 point, and 3-point shots 3 points). If the number of each type of basket is different, and the number of baskets of any two types differs by no more than 4, how many field goals are scored?

A. 11 B. 12 C. 13 D. 14 E. 15

14. Which of the following is a factor of $(10^{2005} + 1)^2 + (10^{2005} + 2)^2 - (10^{2005})^2$?

A. $10^{2005} - 1$ B. $10^{2005} + 3$ C. $10^{2005} + 4$ D. $10^{2005} + 5$ E. $10^{2005} + 6$

15. The volume of cylinder A is 108π, which is twice the volume of cylinder B. If the radius and height of A are the height and radius respectively of B, find the height of cylinder B.

A. 3 B. 4 C. 6 D. 9 E. 12

16. In how many ways can nine identical dominos (2x1 rectangles) be used to exactly cover a 3x6 rectangle with no overlap? Assume two coverings are different if the nine dominos are not in exactly the same positions.

A. 27 B. 31 C. 35 D. 41 E. 47

17. Two triangular regions are formed in the first quadrant, one with vertices (0,0), (5,0), and (0,12), the other with vertices (0,0), (8,0), and (0,6). Find the area to the nearest integer of the region they have in common.

A. 15 B. 17 C. 19 D. 21 E. 23

18. A triangle has sides of length a, b, and c, which are consecutive integers in increasing order, and $\cos C = \frac{5}{16}$. Find cos A.

A. $\frac{5}{8}$ B. $\frac{7}{11}$ C. $\frac{13}{20}$ D. $\frac{2}{3}$ E. $\frac{11}{16}$

19. If $p > 5$ is a prime number, what is the largest integer which must be a factor of $p^4 - 1$?

A. 120 B. 150 C. 180 D. 240 E. 400

20. The circumradius of a triangle is the radius of the circle which contains all three of the triangle's vertices. The length of the circumradius of the triangle with sides of length 193, 194, and 195 is a rational number. Find this length to the nearest tenth.

A. 112.0 B. 112.1 C. 112.2 D. 112.3 E. 112.4

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AMATYC STUDENT MATHEMATICS LEAGUE

NAME: KEY – Spring 2005

COLLEGE: ____________

STATE: ____________

ROUND: 1 **2**

	Student's Responses	Local Corrector
1	**E**	
2	**D**	
3	**B**	
4	**E**	
5	**C**	
6	**Correct For All Students**	C
7	**E**	
8	**B**	
9	**B**	
10	**B**	
11	**E**	
12	**A**	
13	**C**	
14	**D**	
15	**C**	
16	**D**	
17	**C**	
18	**C**	
19	**D**	
20	**A**	

correct = ____________

incorrect = ____________

blank= ____________

$\square$ = # correct $\times$ 2

\- $\square$ = # incorrect $\times \frac{1}{2}$

$\square$ = **score**